Precalculus A

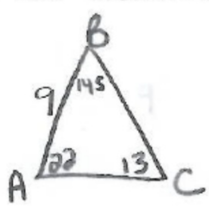
5.6 Law of Cosines

Homework: Finish Practice Worksheet

D. Paulson

Solve each triangle by finding all the missing sides and angles. Round your answer to the nearest tenth.
MAKE SURE TO CHECK FOR AMBIGUOUS CASES.

1. $m\angle C = 13°, m\angle A = 22° c = 9$ AAS - 1 Δ



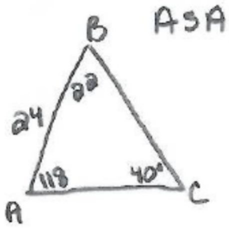
① $m\angle B = 180 - (22+13) = 145$

$\boxed{m\angle B = 145°}$

② $\frac{\sin 13}{9} = \frac{\sin 22}{a}$ $\quad a = \frac{9\sin 22}{\sin 13}$ $\quad \boxed{a = 15.0}$

③ $\frac{\sin 13}{9} = \frac{\sin 145}{b}$ $\quad b = \frac{9 \sin 145}{\sin 13}$ $\quad \boxed{b = 22.9}$

2. $m\angle A = 118°, m\angle B = 22°, c = 24$ ASA - 1 Δ



① $180 - (118 + 22) = 40°$

$\boxed{m\angle C = 40°}$

② $\frac{\sin 40}{24} = \frac{\sin 18}{a}$ $\quad a = \frac{24 \sin 18}{\sin 40}$ $\quad \boxed{a = 33.0}$

③ $\frac{\sin 40}{24} = \frac{\sin 22}{b}$ $\quad b = \frac{24 \sin 22}{\sin 40}$ $\quad \boxed{b = 14.0}$

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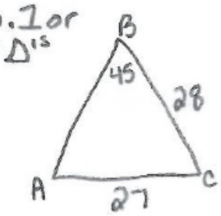
3. $m\angle B = 45°, a = 28, b = 27$

SSA - 0, 1 or 2 Δ's



① $\frac{\sin 45}{27} = \frac{\sin A}{28}$ $\sin A = \frac{28\sin 45}{27}$ $\boxed{m\angle A = 47°}$

② $180-(45+47)=88$ $\boxed{m\angle C = 88°}$

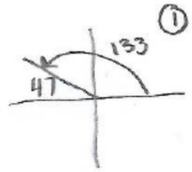
③ $\frac{\sin 45}{27} = \frac{\sin 88}{c}$ $c = \frac{27\sin 88}{\sin 45}$ $\boxed{c = 38.2}$

Check for 2nd Δ

① $180-(133+45)=2$

$\boxed{m\angle A = 133°}$

$\boxed{m\angle C = 2°}$



② $\frac{\sin 45}{27} = \frac{\sin 2}{c}$

$c = \frac{27\sin 2}{\sin 45} = 1.3$

$\boxed{c = 1.3}$

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The Law of Cosines (SAS)

In any triangle ABC with angles A, B, and C and opposite sides a, b and c, respectively, the following equation is true:

$$c^2 = a^2 + b^2 - 2ab(\cos C)$$

$$a^2 = c^2 + b^2 - 2cb(\cos A)$$

$$b^2 = a^2 + c^2 - 2ac(\cos B)$$

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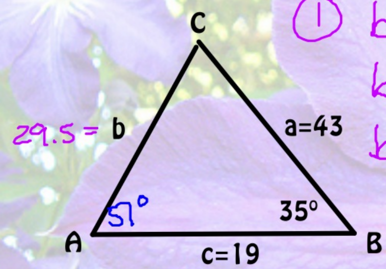
$c^2 = a^2 + b^2 - 2ab(\cos C)$

Example:

Solve the triangle, given B=35°, a=43, and c=19.



① $b^2 = a^2 + c^2 - 2ac(\cos B)$

$b^2 = 43^3 + 19^2 - 2(43)(19)(\cos 35)$

$b^2 = 871.5$

$b = 29.5$

② $\dfrac{\sin 35}{29.5} = \dfrac{\sin A}{43}$

$\sin A = \dfrac{43 \sin 35}{29.5}$

$m\angle A =$

③ $180 - (57 + 35) =$

$m\angle C = 88°$

A= 57°

C= 88°

b= 29.5

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The Law of Cosines (SSS)

SSS

In any triangle ABC with angles A, B, and C and opposite sides a, b and c, respectively, the following equation is true:

$$c^2 = a^2 + b^2 - 2ab(\cos C)$$

Rewrite to cos C =

$c^2 = a^2 + b^2 - 2ab(\cos C)$

$c^2 - a^2 - b^2 = -2ab(\cos C)$

$$\boxed{\dfrac{c^2 - a^2 - b^2}{-2ab} = \cos C}$$

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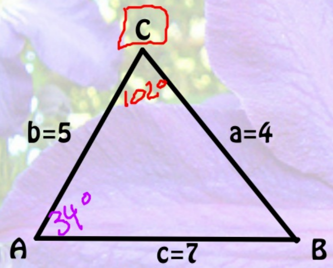
$c^2 = a^2 + b^2 - 2ab(\cos C)$

Example:

Solve the triangle, given a=4, b= 5 and c=7.



$c^2 = a^2 + b^2 - 2ab(\cos C)$

$7^2 = 4^2 + 5^2 - 2(4)(5)(\cos C)$

$\cos C = \dfrac{c^2 - a^2 - b^2}{-2ab}$

$\cos C = \dfrac{7^2 - 4^2 - 5^2}{-2(4)(5)}$

$m\angle C = 102°$

(2) $\dfrac{\sin 102}{7} = \dfrac{\sin A}{4}$

$\sin A = \dfrac{4 \sin 102}{7}$

$m\angle A = 34°$

(3) $180 - (102 + 34)$

$m\angle B = 44°$

A = 34°

B = 44°

C = 102°

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