Precalculus A

5.4 Multiple Angle Identities

Day 1

HW: **Assignment #13, #1-12 only and skip #3**
Assignment #14, #11-12 only

D. Paulson

Double-Angle Identities

$$\sin 2\alpha = 2 \sin\alpha \cos \alpha$$

$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$$

$$= 1 - 2 \sin^2\alpha$$

$$= 2 \cos^2\alpha - 1$$

$$\tan 2\alpha = \frac{2 \tan \alpha}{1 - \tan^2\alpha}$$

D. Paulson

Half Angle Identities

$\sin\frac{\alpha}{2} = \sqrt{\frac{1 - \cos\alpha}{2}}$	
$\cos\frac{\alpha}{2} = \sqrt{\frac{1 + \cos\alpha}{2}}$	$\tan\frac{\alpha}{2} = \frac{\sin\alpha}{1 + \cos\alpha}$ $\tan\frac{\alpha}{2} = \frac{1 - \cos\alpha}{\sin\alpha}$

Examples:

Simplify the following:

1. $2\sin 50^{\circ}\cos 50^{\circ}$

$\sin(2\cdot 50) = \boxed{\sin 100^{\circ}}$

2. $\dfrac{2\tan 40^{\circ}}{1 - \tan^2 40^{\circ}}$

$\tan(2\cdot 40) = \boxed{\tan 80^{\circ}}$

$\sin 2\alpha = 2\sin\alpha\cos\alpha$
$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$ $= 1 - 2\sin^2\alpha$ $= 2\cos^2\alpha - 1$
$\tan 2\alpha = \frac{2\tan\alpha}{1 - \tan^2\alpha}$

$\sin\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}}$	$\tan\frac{\alpha}{2} = \frac{1-\cos\alpha}{\sin\alpha}$
$\cos\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{2}}$	$\tan\frac{\alpha}{2} = \frac{\sin\alpha}{1+\cos\alpha}$

Examples:

Simplify the following:

3. $1 - 2\sin^2 72^\circ$

$\cos(2 \cdot 72) = \boxed{\cos 144^\circ}$

4. $\dfrac{\sin 122^\circ}{1 + \cos 122^\circ}$

$\tan\left(\frac{122}{2}\right) = \boxed{\tan 61^\circ}$

$\sin 2\alpha = 2\sin\alpha\cos\alpha$
$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$ $= 1 - 2\sin^2\alpha$ $= 2\cos^2\alpha - 1$
$\tan 2\alpha = \dfrac{2\tan\alpha}{1 - \tan^2\alpha}$

$\sin\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}}$	$\tan\frac{\alpha}{2} = \frac{1-\cos\alpha}{\sin\alpha}$
$\cos\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{2}}$	$\tan\frac{\alpha}{2} = \frac{\sin\alpha}{1+\cos\alpha}$

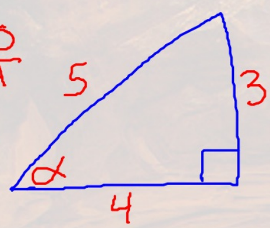
D. Paulson

Given that the tan A = 3/4, find cos 2A.

$\tan\alpha = \frac{O}{A}$

$\cos = \frac{A}{H}$

$\sin = \frac{O}{H}$



$3^2 + 4^2 = x^2$

$9 + 16 = x^2$

$25 = x^2$

$5 = x$

$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$

$= \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2 =$

$= \frac{16}{25} - \frac{9}{25} = \boxed{\frac{7}{25}}$

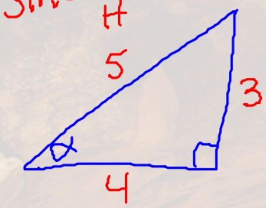
$\sin 2\alpha = 2\sin\alpha\cos\alpha$
$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$ $= 1 - 2\sin^2\alpha$ $= 2\cos^2\alpha - 1$
$\tan 2\alpha = \dfrac{2\tan\alpha}{1 - \tan^2\alpha}$

$\sin\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}}$	$\tan\frac{\alpha}{2} = \frac{1-\cos\alpha}{\sin\alpha}$
$\cos\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{2}}$	$\tan\frac{\alpha}{2} = \frac{\sin\alpha}{1+\cos\alpha}$

D. Paulson

Given that the sin A = 3/5, find tan 2A.

$\sin = \frac{O}{H}$



$\tan = \frac{O}{A}$

$$\tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2 x}$$

$$= \frac{2\left(\frac{3}{4}\right)}{1-\left(\frac{3}{4}\right)^2}$$

$$= \frac{\frac{6}{4}}{1-\frac{9}{16}} = \frac{\frac{3}{2}}{\frac{7}{16}}$$

$$= \frac{3}{1\cancel{2}} \cdot \frac{\cancel{16}\,8}{7} = \boxed{\frac{24}{7}}$$

$\sin 2\alpha = 2\sin\alpha\cos\alpha$	
$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$ $= 1 - 2\sin^2\alpha$ $= 2\cos^2\alpha - 1$	
$\tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha}$	

$\sin\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}}$	$\tan\frac{\alpha}{2} = \frac{1-\cos\alpha}{\sin\alpha}$
$\cos\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{2}}$	$\tan\frac{\alpha}{2} = \frac{\sin\alpha}{1+\cos\alpha}$

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