Precalculus A

5.3 Sum and Difference Identities

Day 1

Homework: Assignment #9

D. Paulson

...nd all solutions to the equation in the interval [0°,360°). You do not need a calculator

1. $2\cos x\sin x - \cos x = 0$

$\cos x(2\sin x - 1) = 0$

$\cos x = 0 \quad 2\sin x - 1 = 0$

$\boxed{90°, 270°} \quad \sin x = \frac{1}{2} \quad \boxed{30°, 150°}$

2. $\sqrt{2}\tan x\cos x - \tan x = 0$

$\tan x(\sqrt{2}\cos x - 1) = 0$

$\tan x = 0 \quad \sqrt{2}\cos x - 1 = 0$

$\boxed{0, 180°} \quad \cos x = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

Ex #1 $\quad \boxed{45°, 315°}$

3. $\tan x\sin^2 x = \tan x$

$\tan x\sin^2 x - \tan x = 0$

$\tan x(\sin^2 x - 1) = 0$

$\tan x(\sin x - 1)(\sin x + 1) = 0$

$\tan x = 0 \quad \sin x = 1 \quad \sin x = -1$

$\boxed{0°, 180°} \quad \boxed{90°} \quad \boxed{270°}$

4. $\sin x\tan^2 x = \sin x$

$\sin x\tan^2 x - \sin x = 0$

$\sin x(\tan^2 x - 1) = 0$

$\sin x(\tan x + 1)(\tan x - 1) = 0$

$\sin x = 0 \quad \tan x = -1 \quad \tan x = 1$

$\boxed{0°, 180°} \quad \boxed{135°, 315°} \quad \boxed{45°, 225°}$

5. $\tan^2 x = 3$

$\tan x = \pm\sqrt{3}$

$\boxed{60°, 120°, 240°, 300°}$

6. $2\sin^2 x = 1$

$\sin^2 x = \frac{1}{2}$

$\sin x = \pm\frac{1}{\sqrt{2}} = \pm\frac{\sqrt{2}}{2}$

$\boxed{45°, 135°, 225°, 315°}$

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Find all solutions to the equation in the interval [0°,360°). You do not need a calculator.

7. $4\cos^2 x - 4\cos x + 1 = 0$

$(2\cos x - 1)(2\cos x - 1) = 0$

$\cos x = \frac{1}{2}$

$\boxed{60^\circ, 300^\circ}$

8. $2\sin^2 x + 3\sin x + 1 = 0$

$(2\sin x + 1)(\sin x + 1) = 0$

$2\sin x + 1 = 0 \qquad \sin x + 1 = 0$

$\sin x = -\frac{1}{2} \qquad \sin x = -1$

$\boxed{210^\circ, 330^\circ} \qquad \boxed{270^\circ}$

9. $\sin^2 x - 2\sin x = 0$

$\sin x(\sin x - 2) = 0$

$\sin = 0 \qquad \sin x = 2$

$\boxed{0^\circ, 180^\circ} \qquad \boxed{DNE}$

10. $3\sin x = 2\cos^2 x$

$3\sin x - 2(1 - \sin^2 x) = 0$

$3\sin x - 2 + 2\sin^2 x = 0$

$2\sin^2 x + 3\sin x - 2 = 0$

$(2\sin x - 1)(\sin x + 2) = 0$

$\sin x = \frac{1}{2} \qquad \sin x = -2$

$\boxed{30^\circ, 150^\circ} \qquad \boxed{DNE}$

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The Sum and Difference Formulas

$$Cos(\alpha + \beta) = Cos\alpha Cos\beta - Sin\alpha Sin\beta$$

$$Cos(\alpha - \beta) = Cos\alpha Cos\beta + Sin\alpha Sin\beta$$

$$Sin(\alpha + \beta) = Sin\alpha Cos\beta + Cos\alpha Sin\beta$$

$$Sin(\alpha - \beta) = Sin\alpha Cos\beta - Cos\alpha Sin\beta$$

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The Sum and Difference Formulas

$$Tan(\alpha + \beta) = \frac{Tan\alpha + Tan\beta}{1 - Tan\alpha Tan\beta}$$

$$Tan(\alpha - \beta) = \frac{Tan\alpha - Tan\beta}{1 + Tan\alpha Tan\beta}$$

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The Sum and Difference Formulas

Sum and Difference Identities
$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$
$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$
$\tan\left(\alpha \pm \beta\right) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$

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Examples:

Write the expression as the sine, cosine, or tangent of an angle.

$\overset{\alpha}{\cos 18^\circ}\overset{\beta}{\cos 42^\circ} - \overset{\alpha}{\sin 18^\circ}\overset{\beta}{\sin 42^\circ}$

$\cos(\alpha+\beta)$

$\cos(18+42) = \cos(60^\circ) = \frac{1}{2}$

$\overset{\alpha}{\sin 18^\circ}\overset{\beta}{\cos 42^\circ} + \overset{\alpha}{\cos 18^\circ}\overset{\beta}{\sin 42^\circ}$

$\sin(\alpha+\beta)$

$\sin(18+42)$

$\sin(60^\circ) = \frac{\sqrt{3}}{2}$

$\overset{\alpha}{\sin 18^\circ}\overset{\beta}{\cos 42^\circ} - \overset{\alpha}{\cos 18^\circ}\overset{\beta}{\sin 42^\circ}$

$\sin(\alpha-\beta)$

$\sin(18-42)$

$\sin(-24^\circ)$

$\cos 18^\circ \cos 42^\circ + \sin 18^\circ \sin 42^\circ$

$\cos(\alpha-\beta)$

$\cos(18-42)$

$\cos(-24^\circ)$

Examples:

Write the expression as the sine, cosine, or tangent of an angle.

$$\frac{\overset{\alpha}{\tan 72^\circ} - \overset{\beta}{\tan 42^\circ}}{1 + \overset{\alpha}{\tan 72^\circ}\,\overset{\beta}{\tan 42^\circ}}$$

$\tan(\alpha-\beta) = \tan(72-42) = \tan(30^\circ) = \frac{\sqrt{3}}{3}$

$$\frac{\tan 25^\circ + \tan 20^\circ}{1 - \tan 25^\circ \tan 20^\circ}$$

$\tan(\alpha+\beta) = \tan(25+20) = \tan(45^\circ) = 1$