AP Statistics

Chapter 6:
Random Variables

Day 8

MC

HW: p. 405-406, ~~#93-99~~ odd, 101-104

FRAPPY

Geometric Random Variables

In a binomial setting, the number of trials n is fixed in advance, and the binomial random variable X counts the number of success. The possible values of X are 0, 1, 2, ..., n.

In other situations, the goal is to repeat a chance process until a success occurs:

- Roll a pair of dice until you get doubles.
- In basketball, attempt a three-point shot until you make one.
- Keep placing a $1 bet on the number 15 in roulette until you win.

Geometric Random Variables

These are all examples of a geometric setting.

Although the number of trials isn't fixed in advance, the trials are independent and the probability of success remains constant.

Geometric Random Variables

A geometric setting arises when we perform independent trials of the same chance process and record the number of trials until a particular outcome occurs.

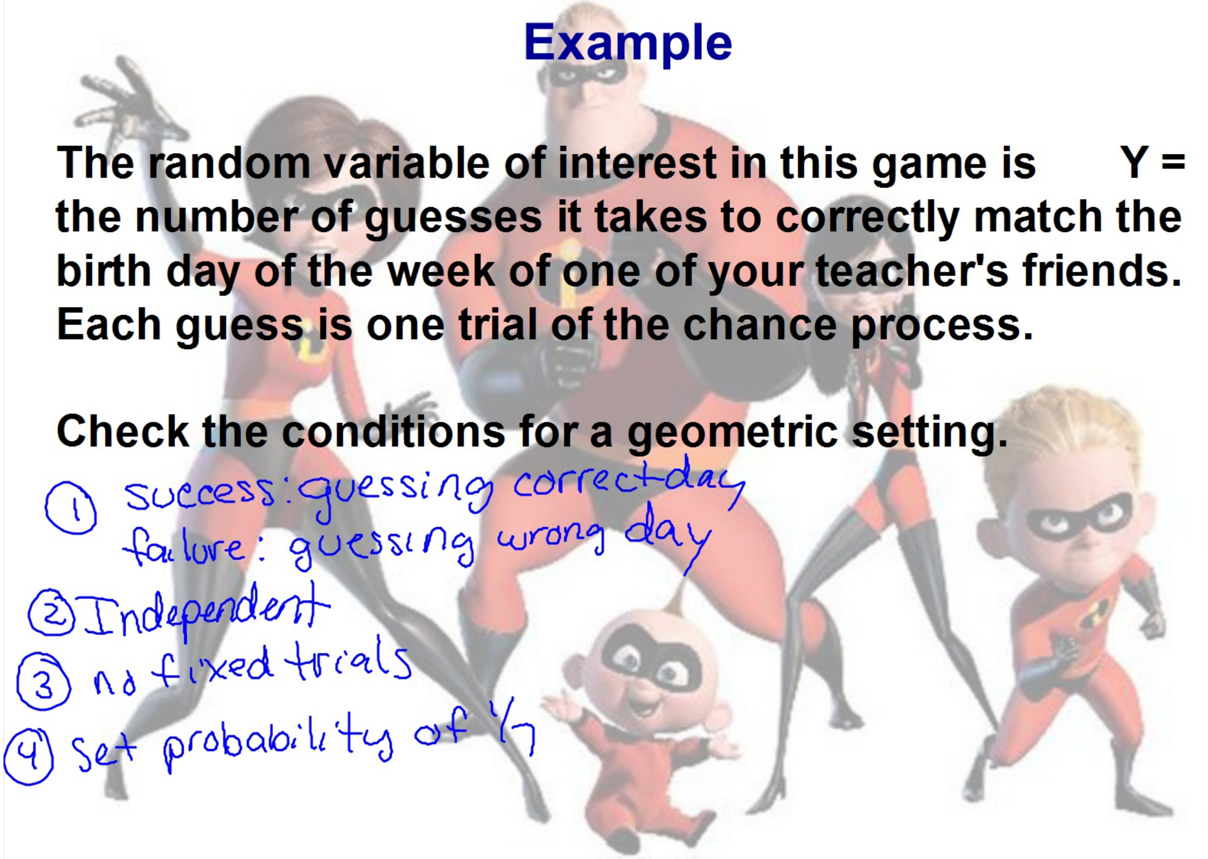
- **The possible outcomes of each trial can be classified as "success" or "failure."**
- **Trials must be independent. Knowing the result of one trial must not have any effect on the result of any other trial.**
- **The goal is to count the number of trials until the first success occurs.**
- **On each trial, the probability p of success must be the same.**

Example

The random variable of interest in this game is Y = the number of guesses it takes to correctly match the birth day of the week of one of your teacher's friends. Each guess is one trial of the chance process.

Check the conditions for a geometric setting.

1. success: guessing correct day
   failure: guessing wrong day
2. Independent
3. no fixed trials
4. set probability of 1/7

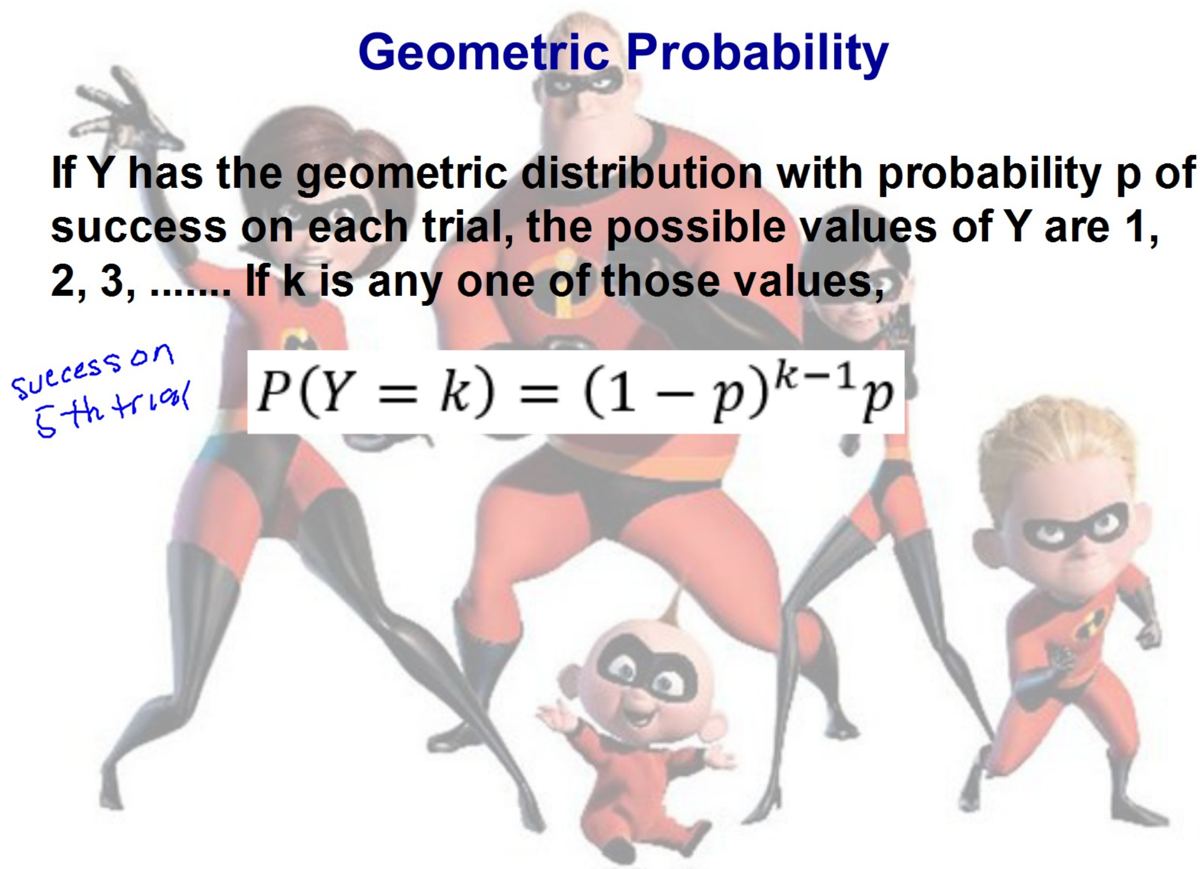


Geometric Probability

If Y has the geometric distribution with probability p of success on each trial, the possible values of Y are 1, 2, 3, ... If k is any one of those values,

success on 5th trial

$$P(Y = k) = (1 - p)^{k-1}p$$



Example

Let's go back to the birthday example. The probability that the student guesses correctly is 1/7. The class currently has only one homework problem. Every time the class guesses incorrectly, they get an additional question for homework.

Find the probability that the the class receives 10 problems as a result of playing the game.

$P(Y=10) = \left(\frac{6}{7}\right)^{10-1}\left(\frac{1}{7}\right) \boxed{= .0357}$

geonompdf$\left(\frac{1}{7}, 10\right) = .0357$

Example

Find $P(Y < 10)$ **and interpret this value in context.**

$P(Y<10) = P(x=1) + P(x=2) \ldots + P(x=9)$

$\left(\frac{6}{7}\right)^{1-1}\left(\frac{1}{7}\right) + \left(\frac{6}{7}\right)^{2-1}\left(\frac{1}{7}\right) \quad \left(\frac{6}{7}\right)^{9-1}\left(\frac{1}{7}\right)$

geonomcdf$\left(\frac{1}{7}, 9\right) \approx .7503$

Mean (Expected Value) of Geometric Random Variables

If Y is a geometric random variable with probability of success p on each trial, then its mean (expected value) is:

$\frac{1}{\frac{1}{7}} = \boxed{7}$

$$E(Y) = \mu_Y = \frac{1}{p}$$

That is, the expected number of trials required to get the first success is 1/p.