

AP Statistics

Chapter 6: Random Variables

Day 7/8

HW: FRAPPY Practice Worksheet

Test Next Tuesday

Binomial Distributions and Sampling

A music distributor inspects an SRS of 10 CDs from a shipment of 10,000 music CDs. Suppose that 10% of the CDs in the shipment are expected to have defective copy-protection schemes that will harm personal computers. Count the number X of bad CDs in the sample.

Why is this technically not a binomial setting?

Not replacing the CD's. Not a constant probability of success.

Binomial Distributions and Sampling

What is $P(X = 0)$ using the binomial distribution?

$$SRS = 10,000$$

$$p = .10$$

$$1 - p = .90$$

$${}^{10000}C_0$$

$$1 \cdot (.10)^0 \cdot (.90)^{10} = .3487$$

What is the actual $P(\text{no defectives})$?

1000 = defective
9000 = non-defective

$$\frac{9000}{10000} \cdot \frac{8999}{9999} \cdot \frac{8998}{9998} \cdots \frac{8991}{9991} =$$

$$.3485$$

Sampling without Replacement Condition

When taking an SRS of size n from a population of size N , we can use a binomial distribution to model the count of successes in the sample as long as:

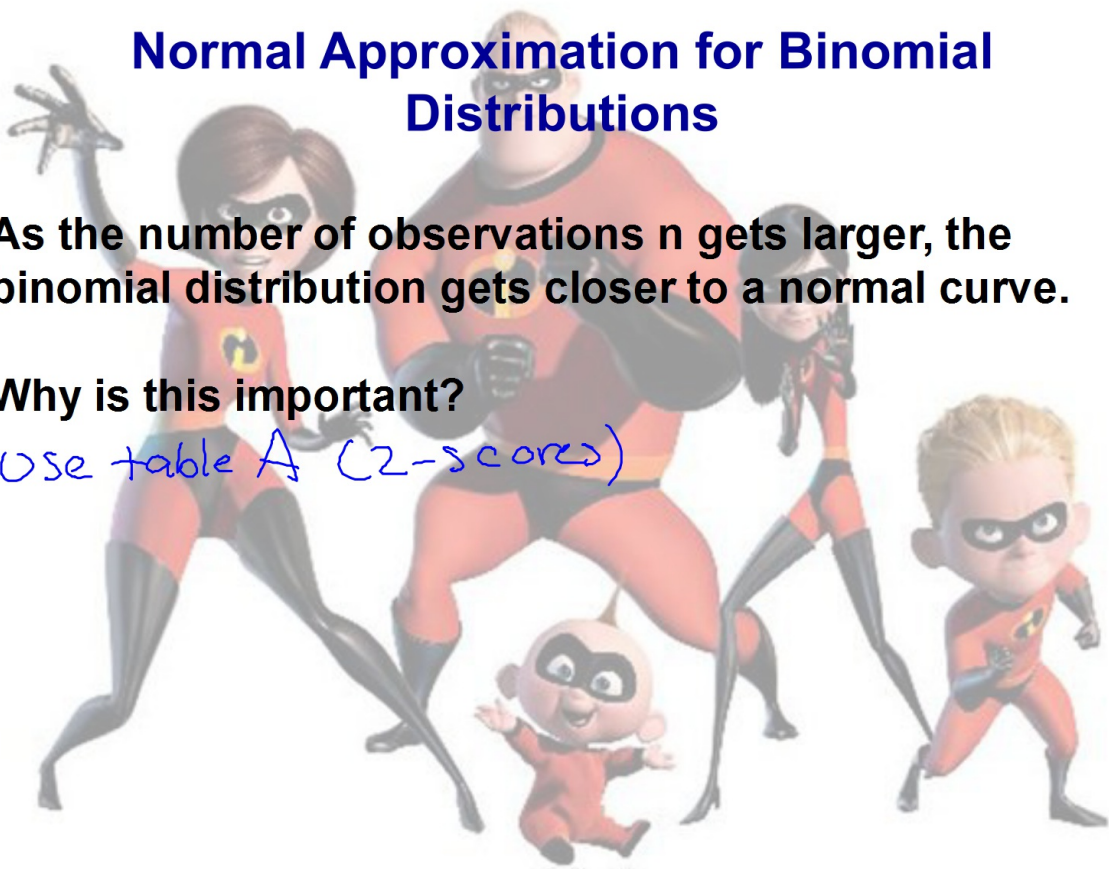
$$n \leq \frac{1}{10} N$$

Normal Approximation for Binomial Distributions

As the number of observations n gets larger, the binomial distribution gets closer to a normal curve.

Why is this important?

Use table A (z-scores)

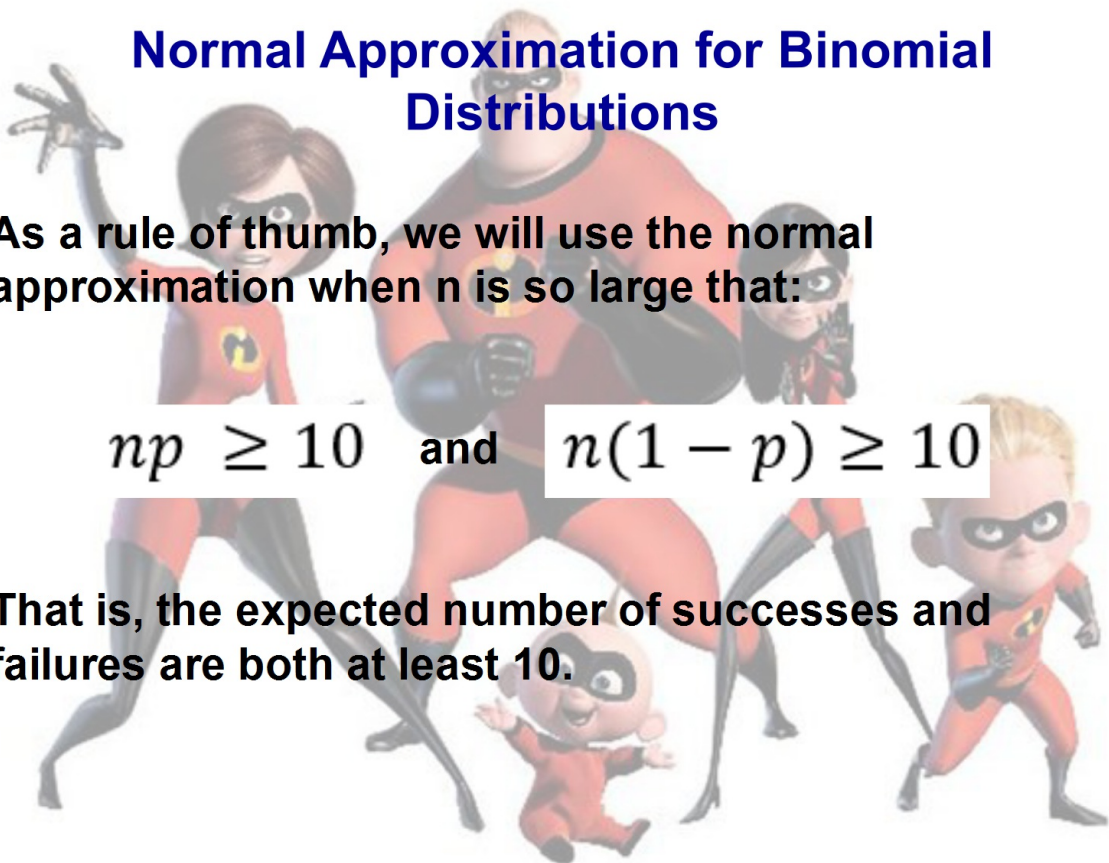


Normal Approximation for Binomial Distributions

As a rule of thumb, we will use the normal approximation when n is so large that:

$$np \geq 10 \quad \text{and} \quad n(1 - p) \geq 10$$

That is, the expected number of successes and failures are both at least 10.



Example

Sample surveys shows that fewer people enjoy shopping than in the past. A survey asked a nationwide random sample of 2500 adults if they agreed or disagreed that "I like buying new clothes, but shopping is often frustrating and time-consuming." The population that the poll wants to draw conclusions about is all U.S. residents aged 18 and over.

Example

Suppose that exactly 60% of all adult U.S residents would say "Agree" if asked the same question. Let X = the number in the sample who agree.

$$n = 2500$$

Check the conditions for using a normal approximation in this setting.

$$\begin{aligned} np &\geq 10 \\ 2500(.60) &\geq 10 \\ 1500 &\geq 10 \end{aligned}$$

$$\begin{aligned} n(1-p) &\geq 10 \\ 2500(.40) &\geq 10 \\ 1000 &\geq 10 \end{aligned}$$

* when it does not say normal distribution.

Example

Use a normal distribution to estimate the probability that 1520 or more of the sample agree.

$$P(X \geq 1520)$$

$$Z = \frac{1520 - 1500}{24.49} = .82$$

$$P(Z \geq .82) = 1 - .7939 = \boxed{.2061}$$

$$\begin{aligned}\mu_x &= 2500(.60) = 1500 \\ \sigma_x &= \sqrt{2500(.60)(.40)} = \\ & 24.49\end{aligned}$$

