AP Statistics

Chapter 6:
Random Variables

Day 7/8

HW: FRAPPY Practice Worksheet

Test Next Tuesday

Binomial Distributions and Sampling

A music distributor inspects an SRS of 10 CDs from a shipment of 10,000 music CDs. Suppose that 10% of the CDs in the shipment are expected to have defective copy-protection schemes that will harm personal computers. Count the number X of bad CDs in the sample.

Why is this technically not a binomial setting?

Not replacing the CD's. Not a constant probability of success.

Binomial Distributions and Sampling

What is P(X = 0) using the binomial distribution?

$1(.10)^0(.90)^{10} = \boxed{.3487}$

SRS = 10,000

p = .10

1 − p = .90

1000 nCr 0

What is the actual P(no defectives)?

1000 = defective

9000 = non-defective

$\frac{9000}{10000} \cdot \frac{8999}{9999} \cdot \frac{8998}{9998} \cdots \frac{8991}{9991} = \boxed{.3485}$

Sampling without Replacement Condition

When taking an SRS of size n from a population of size N, we can use a binomial distribution to model the count of successes in the sample as long as:

$$n \leq \frac{1}{10}N$$

Normal Approximation for Binomial Distributions

As the number of observations n gets larger, the binomial distribution gets closer to a normal curve.

Why is this important?

Use table A (z-scores)

Normal Approximation for Binomial Distributions

As a rule of thumb, we will use the normal approximation when n is so large that:

$$np \geq 10 \quad \text{and} \quad n(1-p) \geq 10$$

That is, the expected number of successes and failures are both at least 10.

Example

Sample surveys shows that fewer people enjoy shopping than in the past. A survey asked a nationwide random sample of 2500 adults if they agreed or disagreed that "I like buying new clothes, but shopping is often frustrating and time-consuming." The population that the poll wants to draw conclusions about is all U.S. residents aged 18 and over.

Example

Suppose that exactly 60% of all adult U.S residents would say "Agree" if asked the same question. Let X = the number in the sample who agree.

$n = 2500$

Check the conditions for using a normal approximation in this setting.

$np \geq 10$ $\quad$ $n(1-p) \geq 10$

$2500(.60) \geq 10$ $\quad$ $2500(.40) \geq 10$

$1500 \geq 10$ $\quad$ $1000 \geq 10$

* when it does not say normal distribution.

Example

Use a normal distribution to estimate the probability that 1520 or more of the sample agree.

$\mu_x = 2500(.60) = 1500$

$\sigma_x = \sqrt{2500(.60)(.40)} = 24.49$

$P(X \geq 1520)$

$z = \frac{1520-1500}{24.49} = .82$

$P(z \geq .82) = 1 - .7939 = \boxed{.2061}$