AP Statistics

Chapter 6:
Random Variables

Day 6

HW: p. 403-404, #69-77 odd

Binomial Random Variables

A random variable X is called a <u>binomial random variable</u> if it meets the following conditions:

- **There are a fixed number of trials, n, of a random phenomenon.**
- **There are only two possible outcomes on each trial, a success or a failure.**
- **The probability of success, p, is constant for each trial.**
- **Each trial is independent of the other trials.**

Examples

Here are three scenarios involving chance behavior. In each case, determine whether the given random variables has a binomial distribution.

- Genetics says that children receive genes from their parents independently. Each child of a particular pair of parents has probability 0.25 of having blood type O. Suppose these parents have 5 children. Let X = the number of children with type O blood.

Examples

- Shuffle a deck of cards. Turn over the first 10 cards, one at a time. Let Y = the number of aces you observe.

- Shuffle a deck of cards. Turn over the top card. Put the card back in the deck, and shuffle again. Repeat this process until you get an ace. Let W = the number of cards required.

Calculating Binomial Probabilities

Each child of a particular pair of parents has probability 0.25 of having type O blood. Genetics says that children receive genes from each of their parents independently. If these parents have 5 children, the count X of children with type O blood is a binomial random variable with n = 5 trials and probability p = 0.25 of a success on each trial. In this setting, a child with type O blood is a "success" (S) and a child with another blood type is a "failure" (F).

Calculating Binomial Probabilities

- What is $P(X = 0)$?
- What is $P(X = 1)$?
- What is $P(X = 2)$?

Binomial Coefficient

The binomial coefficient is the number of ways of arranging n observations.

$$\binom{n}{k} = \frac{n!}{k!\,(n-k)!}$$

n = number of observations

k = number of successes

Calculating Binomial Probabilities

- What is $P(X = 2)$?

Binomial Probability

$$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$$

n = number of observations

k = number of successes

p = probability of success

Example

Each child of a particular pair of parents has probability 0.25 of having type O blood. Suppose the parents have 5 children.

Find the probability that exactly 3 of the children have type O blood.

Example

Each child of a particular pair of parents has probability 0.25 of having type O blood. Suppose the parents have 5 children.

Should the parents be surprised if more than 3 of their children have type O blood? Justify your answer.

Example

When rolling two dice, the probability of rolling doubles is 1/6. Suppose that a game player rolls the dice 4 times, hoping to roll doubles.

Find the probability that the player gets doubles twice in 4 attempts.

$n = 4$ (# of trials)

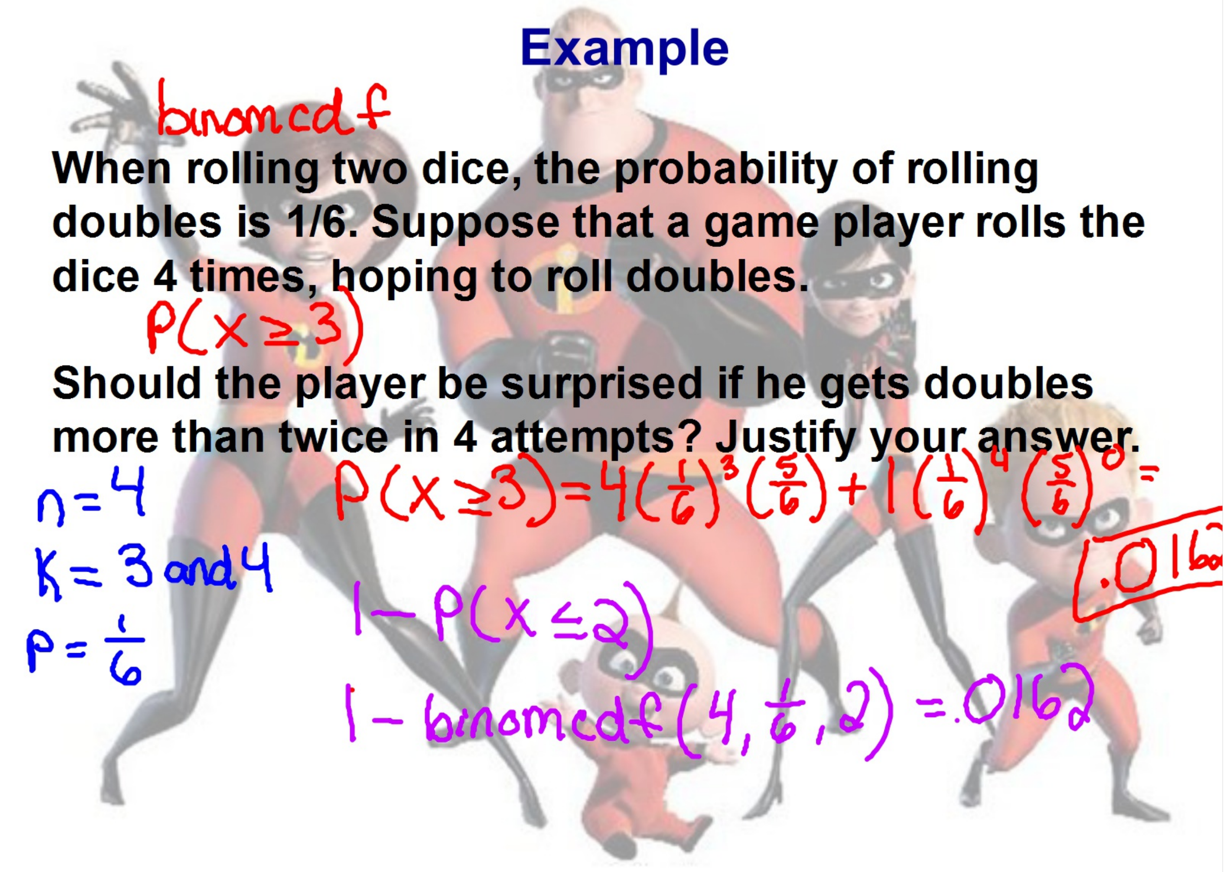
$k = 2$ (# of successes)

$p = \frac{1}{6}$

$P(X=2) = 6\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^2 = .1157$

binompdf(4, 1/6, 2) = .1157

Example



binomcdf

When rolling two dice, the probability of rolling doubles is 1/6. Suppose that a game player rolls the dice 4 times, hoping to roll doubles.

$P(x \geq 3)$

Should the player be surprised if he gets doubles more than twice in 4 attempts? Justify your answer.

$n = 4$

$k = 3$ and 4

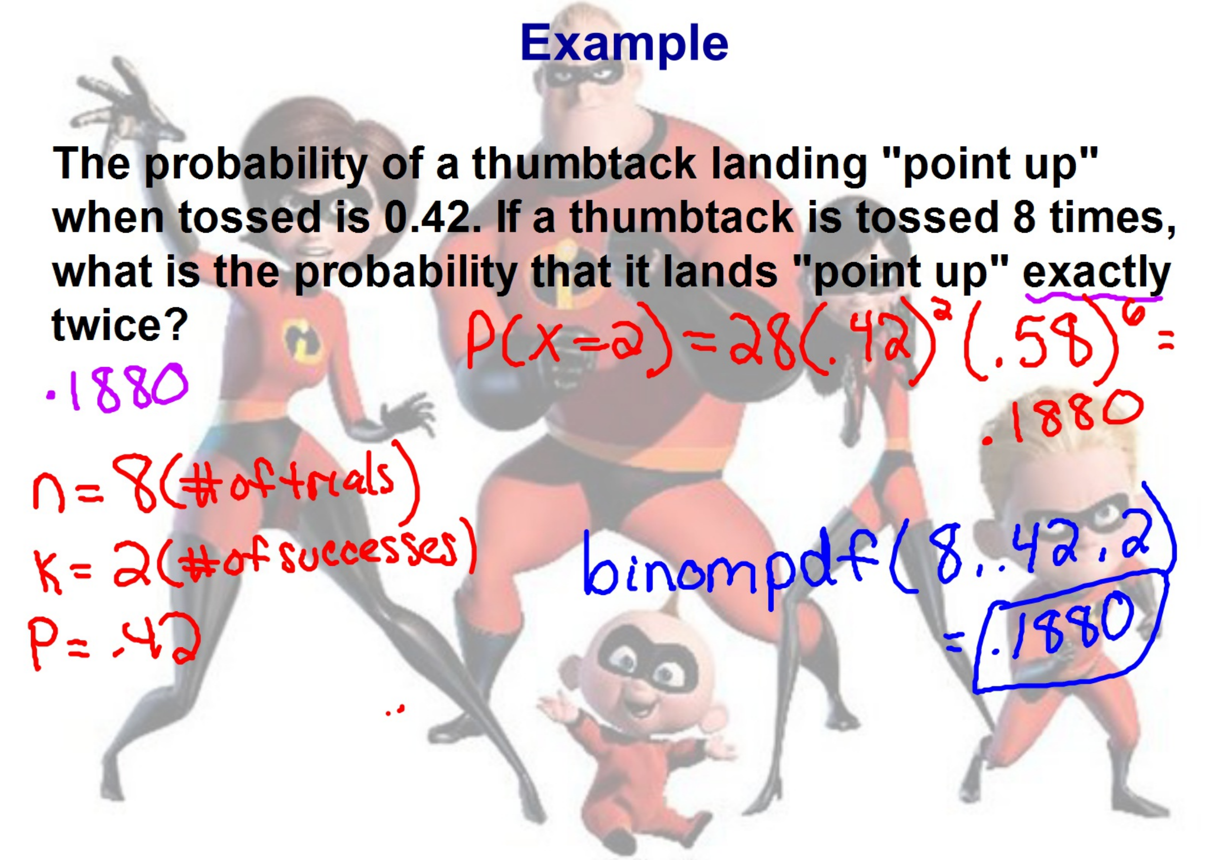
$p = \frac{1}{6}$

$P(x \geq 3) = 4\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right) + 1\left(\frac{1}{6}\right)^4\left(\frac{5}{6}\right)^0 =$ .0162

$1 - P(x \leq 2)$

$1 - \text{binomcdf}(4, \frac{1}{6}, 2) = .0162$

Example



The probability of a thumbtack landing "point up" when tossed is 0.42. If a thumbtack is tossed 8 times, what is the probability that it lands "point up" exactly twice?

.1880

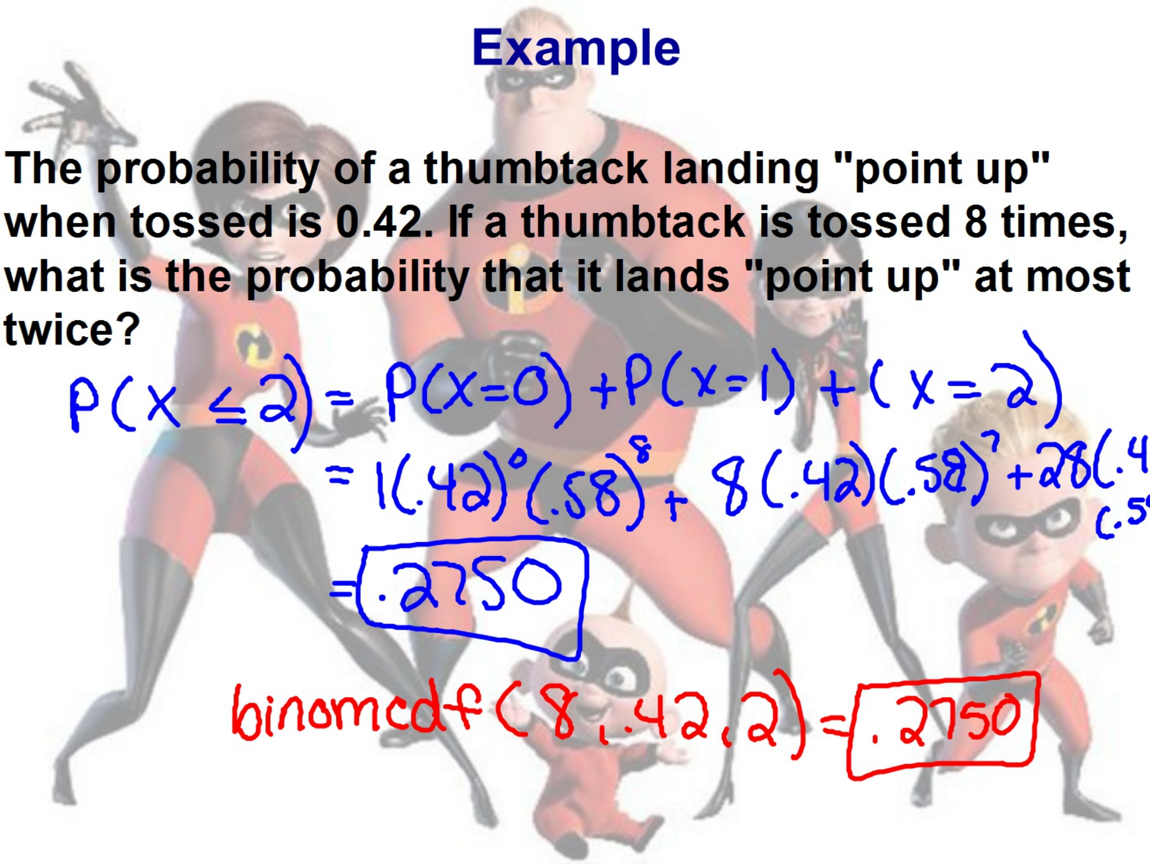
$P(x = 2) = 28(.42)^2(.58)^6 =$ .1880

$n = 8$ (# of trials)

$k = 2$ (# of successes)

$p = .42$

binompdf(8, .42, 2) = .1880



Example

The probability of a thumbtack landing "point up" when tossed is 0.42. If a thumbtack is tossed 8 times, what is the probability that it lands "point up" at most twice?

$P(X \le 2) = P(X=0) + P(X=1) + (X=2)$

$= 1(.42)^0(.58)^8 + 8(.42)(.58)^7 + 28(.42)^2(.58)^6$

$= .2750$

binomcdf(8, .42, 2) = .2750