AP Statistics

Chapter 6:
Random Variables

Day 6

HW: p. 403-404, #69-77 odd

Binomial Random Variables

A random variable X is called a binomial random variable if it meets the following conditions:

- There are a fixed number of trials, n, of a random phenomenon.
- There are only two possible outcomes on each trial, a success or a failure.
- The probability of success, p, is constant for each trial.
- Each trial is independent of the other trials.

Examples

Here are three scenarios involving chance behavior. In each case, determine whether the given random variables has a binomial distribution.

- Genetics says that children receive genes from their parents independently. Each child of a particular pair of parents has probability 0.25 of having blood type O. Suppose these parents have 5 children. Let X = the number of children with type O blood.

yes

Examples

- Shuffle a deck of cards. Turn over the first 10 cards, one at a time. Let Y = the number of aces you observe.

No.

- Shuffle a deck of cards. Turn over the top card. Put the card back in the deck, and shuffle again. Repeat this process until you get an ace. Let W = the number of cards required.

No

Calculating Binomial Probabilities

Each child of a particular pair of parents has probability 0.25 of having type O blood. Genetics says that children receive genes from each of their parents independently. If these parents have 5 children, the count X of children with type O blood is a binomial random variable with n = 5 trials and probability p = 0.25 of a success on each trial. In this setting, a child with type O blood is a "success" (S) and a child with another blood type is a "failure" (F).

Calculating Binomial Probabilities

- **What is P(X = 0)?** FFFFF $= (.75)^5 = \boxed{.2373}$
- **What is P(X = 1)?** $(.25)(.75)^4 + (.75)(.25)(.75)^3 + (.75)^2(.25)(.75)^2 + (.75)^3(.25)(.75$ $+ (.75)^4(.25) = \boxed{.3955}$

SFFFF
FSFFF
FFSFF
FFFSF
FFFFS

- **What is P(X = 2)?** $(.25)^2(.75)^3(10) = \boxed{.2637}$

10 combinations

Binomial Coefficient

→ # of combinations

The <u>binomial coefficient</u> is the number of ways of arranging n observations.

$\binom{5}{2} = \frac{5!}{2!(5-2)!}$

$$\binom{n}{k} = \frac{n!}{k!\,(n-k)!}$$

n = number of observations

k = number of successes

Calculating Binomial Probabilities

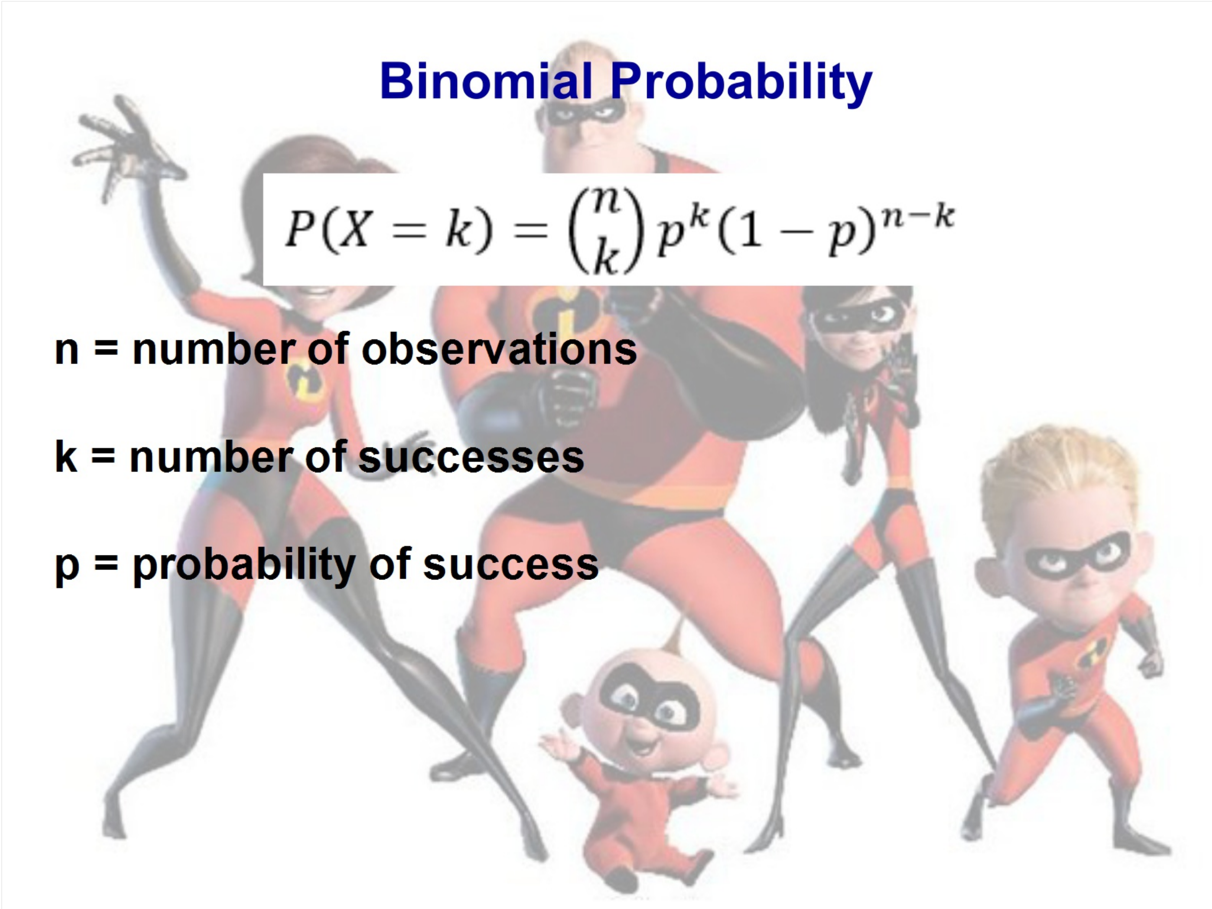
- What is P(X = 2)?

$n = 5$ (total children)

$k = 2$ (success)

$5 \, {}_nC_r \, 2 = 10$

$P(X=2) = 10(.25)^2(.75)^3 = \boxed{.2637}$



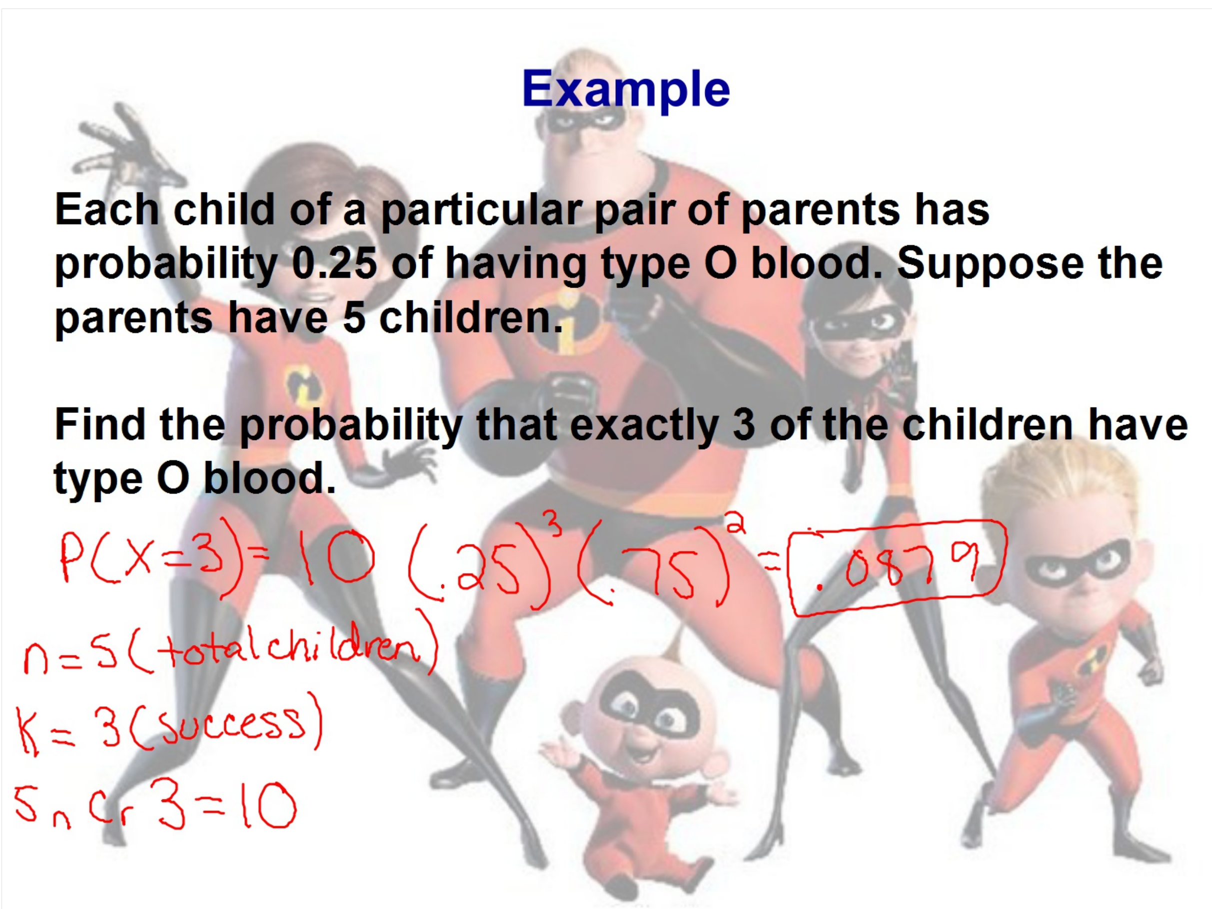
Binomial Probability

$$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$$

n = number of observations

k = number of successes

p = probability of success



Example

Each child of a particular pair of parents has probability 0.25 of having type O blood. Suppose the parents have 5 children.

Find the probability that exactly 3 of the children have type O blood.

$P(X=3) = 10\,(.25)^3(.75)^2 = \boxed{.0879}$

n = 5 (total children)

k = 3 (success)

$5\ {}_nC_r\ 3 = 10$

Example

Each child of a particular pair of parents has probability 0.25 of having type O blood. Suppose the parents have 5 children.

Should the parents be surprised if more than 3 of their children have type O blood? Justify your answer.

$P(X=4)+P(X=5) = 5(.25)^4(.75) + 1(.25)^5(.75)^0$

$5 \, nCr \, 4 = 5$ $\boxed{= .0156}$

$5 \, nCr \, 5 = 1$ $\boxed{\text{Yes!}}$