AP Statistics

Chapter 8:
Estimating with Confidence

Day 5

HW: Lesson 5 Practice Worksheet

How To Find Mean Confidence Intervals When σ Is Unknown

We always like to use $SE_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$ when possible, but we very rarely know the population standard deviation.

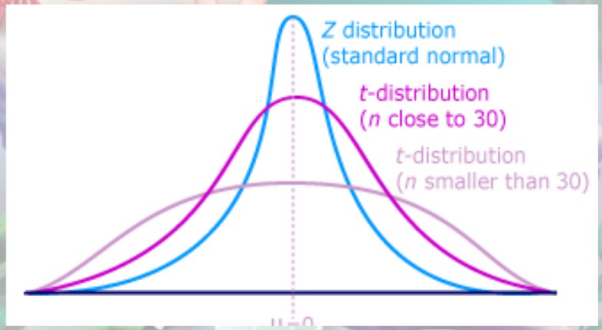
Therefore, we have to use the sample standard deviation.

$$SE_{\bar{x}} = \frac{s}{\sqrt{n}}$$

How To Find Mean Confidence Intervals When σ Is Unknown

Since we are no longer using the population standard deviation, we can no longer use z*.

When we use the sample standard deviation, we use a t-distribution with a t*.



How To Find Mean Confidence Intervals When σ Is Uknown

Standard Error: $SE_{\bar{x}} = \frac{s}{\sqrt{n}}$

Confidence Interval: $CI = \bar{x} \pm t * \frac{s}{\sqrt{n}}$

The t-Statistic

n=30 df=29
90% $t^* = 1.699$

The t-statistic has the same interpretation as any s

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Table B — t distribution critical values

	Tail probability p											
df	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6
2	.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60
3	.765	.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92
4	.741	.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	.727	.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	.718	.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	.711	.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	.706	.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	.703	.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	.700	.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	.697	.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	.695	.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	.694	.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	.692	.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	.691	.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	.690	.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	.689	.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	.688	.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.611	3.922
19	.688	.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	.687	.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850
21	.686	.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	.686	.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	.685	.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	.685	.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	.684	.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	.684	.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	.684	.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28	.683	.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	.683	.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	.683	.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
40	.681	.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	.679	.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496

The Conditions for Mean Confidence Intervals

- Is it a random sample?
- 10% condition to use the standard error formula.
- Is the distribution normal or n ≥ 30 to use the t* scores.

Example

* check conditions

A survey was conducted involving 250 out of 125,000 families living in the city. The average amount of income tax paid per family of the sample was $3540 with a standard deviation of $1150. Establish a 99% confidence interval for the total taxes paid by all the families in the city.

$n = 250$

$\bar{x} = 3540$

$SE = \frac{1150}{\sqrt{250}} = 72.73$

$df = 249$

$CL = 99\%$

$t^* = 2.626$

By hand

$t^* = 2.596$

calculator

$CI = \bar{x} \pm t^* SE$

$3540 \pm 2.626(72.73)$

$(3349.01, 3730.99)$

* calc

$(3351.19, 3728.81)$

$125{,}000(3351.19, 3728.81)$

$(418{,}898{,}750, 466{,}101{,}250)$

I am 99% confident that the average total taxes paid by all families in the city is between 418,898,750 and 466,101,250.

Table B t distribution critical values

Example

As part of their final AP project, Christina and Rachel randomly selected 18 rolls of a generic brand of toilet paper to measure how well this brand could absorb water. To do this, they poured 1/4 cup of water onto a hard surface and counted how many squares it took to completely absorb the water. Here are their results:

29	20	25	29	21	24	27	25	24
29	24	27	28	21	25	26	22	23

Construct and interpret a 95% confidence interval for the mean number of squares of generic toilet paper needed to absorb 1/4 cup of water.

Table B t distribution critical values

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