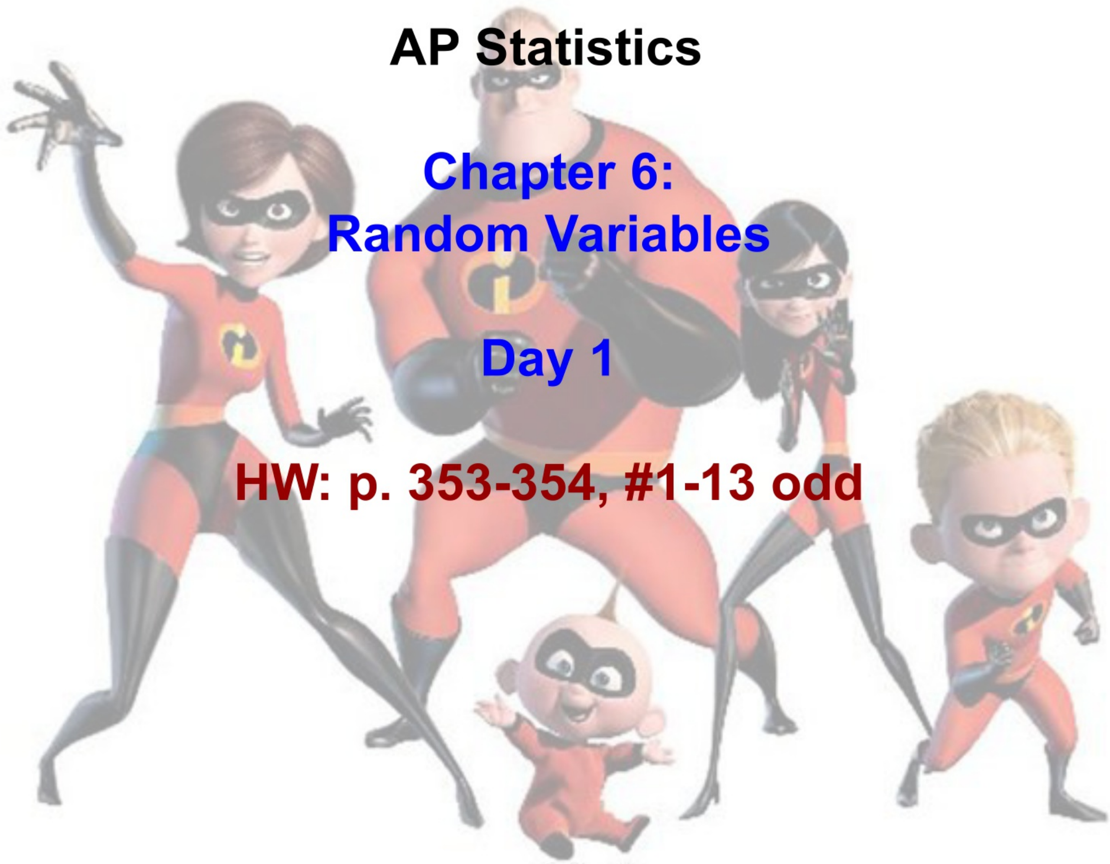
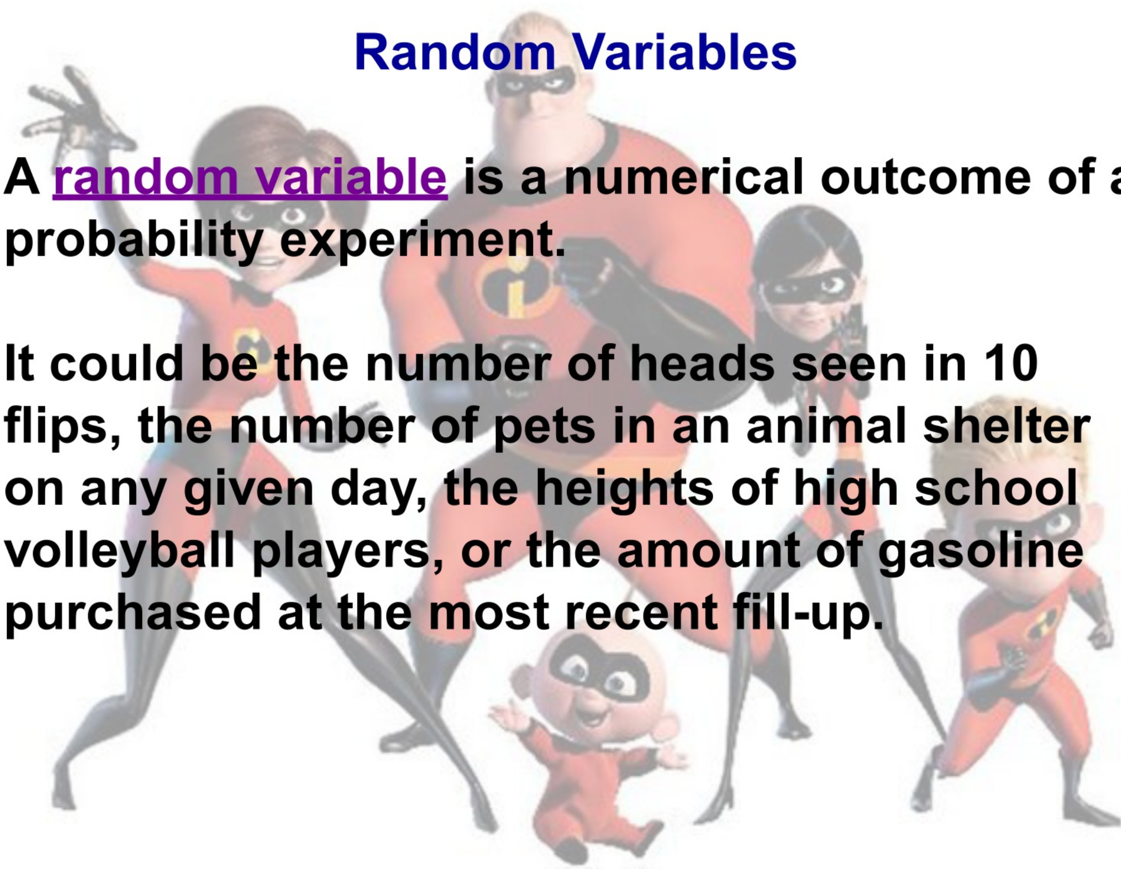


AP Statistics

Chapter 6:
Random Variables

Day 1

HW: p. 353-354, #1-13 odd



Random Variables

A random variable is a numerical outcome of a probability experiment.

It could be the number of heads seen in 10 flips, the number of pets in an animal shelter on any given day, the heights of high school volleyball players, or the amount of gasoline purchased at the most recent fill-up.

Random Variables

Random variables can be either discrete or continuous.

Discrete random variables are those usually obtained by counting, such as counting heads of coin flips or pets in a shelter.

Continuous random variables are those typically found by measuring, such as heights of volleyball players or amount of gasoline.

Discrete Random Variables

The probability distribution of a discrete random variable X is a table, list, graph, or formula giving all possible values taken by a random variable and their corresponding probabilities.

1/6 $\boxed{\text{uniform}}$

1 2 3 4 5 6

1	2	3	4	5	6
1/6	1/6	1/6	1/6	1/6	1/6

Let X be a random variable taking values x_1, x_2, ...x_n, with respective probabilities $P(x_1)$, $P(x_2)$, ...$P(x_n)$.

- **Then $\{(x_1, P(x_1)), (x_2, P(x_2)), ...(x_n, P(x_n))$**

Discrete Random Variables

For a probability distribution to be valid, it must meet two characteristics:

- **For each value of x_i, where i = 1, 2, ..., n, $0 \le P(x_i) \le 1$.**
- $\sum_{i=1}^{n} P(x_i)$ **= 1**

Example

The number of cars X owned by the families of students at a small high school can be given by the probability distribution.

Discrete random variable

Number of cars, x	0	1	2	3	4
Probability (P(x)	0.05	0.15	0.35	0.30	?

What is the probability that a randomly selected student's family owns four cars?

$1-(.05+.15+.35+.30)=\boxed{.15}$

Example

Discrete random variable

Number of cars, x	0	1	2	3	4
Probability (P(x))	0.05	0.15	0.35	0.30	0.15

Create a probability histogram for the random variable X.

Example

In 1952, Dr. Virginia Apgar suggested five criteria for measuring a baby's health at birth: skin color, heart rate, muscle tone, breathing, and response when stimulated. She developed a 0-1-2 scale for each of the five criteria and then totaled the five scores to give the Apgar score which is still in use today. The table below gives the probability distribution for X = Apgar scores of randomly selected baby one minute after birth.

Value:	0	1	2	3	4	5	6	7	8	9	10
Probability:	0.001	0.006	0.007	0.008	0.012	0.020	0.038	0.099	0.319	0.437	0.053

Example

Value:	0	1	2	3	4	5	6	7	8	9	10
Probability:	0.001	0.006	0.007	0.008	0.012	0.020	0.038	0.099	0.319	0.437	0.053

Show that the probability distribution for X is legitimate.

.001 + .006 + .007 + .008 + .012 + .020 + .038 + .099 + .319 + .437 + .053 = $\boxed{1}$

Doctors decided that Apgar scores of 7 or higher indicate a healthy baby. What is the probability that a randomly selected baby is healthy?

$P(X \geq 7) = .099 + .319 + .437 + .053 = \boxed{.908}$

There is a 90.8% chance that a randomly selected baby is born healthy.

Mean (Expected Value)

mu

The mean u_x of random variable X is referred to as the
* **expected value of X and symbolized E(X).**

"average"

The expected value can be found by using this formula:

random variable

$$E(X) = u_x = \sum x_i p_i$$

expected value

mean

probability of random variable

Example

Value:	0	1	2	3	4	5	6	7	8	9	10
Probability:	0.001	0.006	0.007	0.008	0.012	0.020	0.038	0.099	0.319	0.437	0.053

Compute the mean (expected value) of the random variable X and interpret this value in context.

$$\sum x_i p_i = 0(.001) + 1(.006) \ldots + 10(.053) = \boxed{= 8.128}$$

The mean Apgar score of a baby born is 8.128.

Example

On an American roulette wheel, there are 38 slots numbered 1 through 36, plus 0 and 00. Half of the slots from 1 to 36 are red and the other half are black. Both the 0 and 00 slots are green. Suppose that a player places a simple $1 bet on red. If the ball lands in a red slot, the player gets the original dollar back, plus an additional dollar for winning the bet. If the ball lands in a different colored slot, the player loses the dollar bet to the casino.

Let's define the random variable X = net gain from a single $1 bet on red. The possible values of X are -$1 and $1. (The player either gains a dollar or loses a dollar.) What are the probabilities? The chance that the ball lands in a red slot is 18/38. The chance that the ball lands in a different-colored slot is 20/38.

Example

Here is the probability distribution of X:

discrete random variables

Value:	-\$1	\$1
Probability:	20/38	18/38

What is the player's average gain?

$$\sum x_i p_i = (-1)\left(\frac{20}{38}\right) + (1)\left(\frac{18}{38}\right) = -.05$$

On average, you can expect to lose \$.05 per game.

Example

The number of cars X owned by the families of students at a small high school can be given by the probability distribution.

Number of cars, x	0	1	2	3	4
Probability (P(x)	0.05	0.15	0.35	0.30	?

What is the expected value of the probability distribution? 2.35

The mean # of cars owned by the families of students at a small high school is 2.35 cars.