AP Statistics

Chapter 2: Modeling Distributions of Data

Day 1

HW: p. 105-107, #1, 5, 7, 9, 11, 15

Measuring Position

Statisticians often talk about the position of a value, relative to other values in a set of observations.

The most common measures of position are:

- Percentiles
- Quartiles
- Standardized scores (aka, z-scores)

Percentiles

Percentiles divide the data set into 100 equal parts.

An observation at the n^{th} percentile is higher than n percent of all observations.

- If you took a standardized test and your score was in the 83rd percentile, 83% of the people that took that same test scored lower than you.

Quartiles

Quartiles divide the data into four equally sized parts.

The main quartiles are the first quartile, median and third quartile.

- Each quartile contains 25% of the observations no matter how spread out the data are.

Example

Here are the scores of all 25 students in Mr. Morgan's BC Calculus class on their first test.

79 81 80 77 73 83 74 93 78 80 75 67 73
77 83 **86** 90 79 85 83 89 84 82 77 72

The red score is Jenny's score. How did she perform on this test relative to her classmates? (Hint: This means find the percentile.)

$\frac{21}{25} = .84$

Jenny scored in the 84% percentile relative to her classmates.

Example

79 81 80 77 73 83 74 93 78 80 75 67 73
77 83 86 90 79 85 83 89 84 82 77 72

Find the percentile for the following students:

- Norman, who earned a 72.

  $\frac{1}{25} = 4^{th}$ percentile

- Katie, who scored a 93.

  $\frac{24}{25} = 96^{th}$

- The two students who earned scores of 80.

  $\frac{12}{25} = 48^{th}$ percentile.

Standardized Scores (z-scores)

z-scores measure how many standard deviations away from the mean (in either direction) an observation is.

A z-score is calculated by the formula:

$$z = \frac{observation - mean}{standard\ deviation}$$

Interpreting z-scores

- A negative z-score represents an observation less than the mean.
- A positive z-score represents an observation greater than the mean.
- A z-score equal to 0 represents an observation equal to the mean.
- A z-score equal to 1 represents an observation that is 1 standard deviation greater than the mean; a z-score equal to 2, 2 standard deviations greater than the mean; etc.
- A z-score equal to -1 represents an observation that is 1 standard deviation less than the mean; a z-score equal to -2, 2 standard deviations less than the mean; etc.

Example

Where does Jenny's score fall relative to the mean of this distribution?

79 81 80 77 73 83 74 93 78 80 75 67 73
77 83 **86** 90 79 85 83 89 84 82 77 72

$\bar{x} = 80$

$S_x = 6.07$

$z = \frac{86-80}{6.07} = .99$

Jenny's score is .99 standard deviations above the mean.

Example

79 81 80 77 73 83 74 93 78 80 75 67 73
77 83 86 90 79 85 83 89 84 82 77 72

Find the standardized scores for the following students:

- Norman, who earned a 72.

  -1.32 Norman's score is 1.32 standard deviations below the mean

- Katie, who scored a 93.

  2.14 Katie's score is 2.14 standard deviations above the mean.

Computer Printout of Summary Statistics

You will sometimes be given a computer printout summarizing a set of data.

Below is a sample computer printout.

# of observations

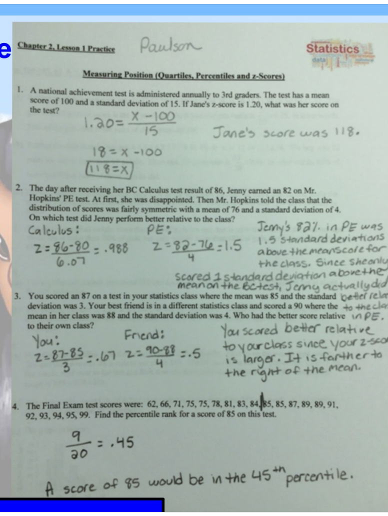
N	Mean	Median	TrMean	StDev	SEMean
80	4.58	4.04	4.37	2.99	0.33

Min	Max	Q1	Q3
0.46	13.63	2.15	6.56

TrMean = Trimmed mean (Usually the mean of the middle 90% of the observations. Potential outliers have been "trimmed" off.)

SEMean = Standard error of the mean

Comple ... ksheet


Chapter 2, Lesson 1 Practice Paulson Statistics

Measuring Position (Quartiles, Percentiles and z-Scores)

1. A national achievement test is administered annually to 3rd graders. The test has a mean score of 100 and a standard deviation of 15. If Jane's z-score is 1.20, what was her score on the test?

$1.20 = \frac{x-100}{15}$ Jane's score was 118.

$18 = x - 100$

$\boxed{118 = x}$

2. The day after receiving her BC Calculus test result of 86, Jenny earned an 82 on Mr. Hopkins' PE test. At first, she was disappointed. Then Mr. Hopkins told the class that the distribution of scores was fairly symmetric with a mean of 76 and a standard deviation of 4. On which test did Jenny perform better relative to the class?

Calculus: $z = \frac{86-80}{6.07} = .988$

PE: $z = \frac{82-76}{4} = 1.5$

Jenny's 82% in PE was 1.5 standard deviations above the mean score for the class. Since she only scored 1 standard deviation above the mean on the BC test, Jenny actually did better relative to the class in PE.

3. You scored an 87 on a test in your statistics class where the mean was 85 and the standard deviation was 3. Your best friend is in a different statistics class and scored a 90 where the mean in her class was 88 and the standard deviation was 4. Who had the better score relative to their own class?

You: $z = \frac{87-85}{3} = .67$

Friend: $z = \frac{90-88}{4} = .5$

You scored better relative to your class since your z-score is larger. It is farther to the right of the mean.

4. The Final Exam test scores were: 62, 66, 71, 75, 75, 78, 81, 83, 84, 85, 85, 87, 89, 89, 91, 92, 93, 94, 95, 99. Find the percentile rank for a score of 85 on this test.

$\frac{9}{20} = .45$

A score of 85 would be in the 45th percentile.